

Model Predictive Control-based Energy Management Strategy for a Series Hybrid Electric Tracked Vehicle

Hong Wang, Yanjun Huang, Amir Khajepour, Qiang Song

Abstract—The series hybrid electric tracked bulldozer (HETB)'s fuel economy heavily depends on its energy management strategy. This paper presents a model predictive controller (MPC) to solve the energy management problem in an HETB for the first time. A real typical working condition of the HETB is utilized to develop the MPC. The results are compared to two other strategies: a rule-based strategy and a dynamic programming (DP) based one. The latter is a global optimization approach used as a benchmark. The effect of the MPC's parameters (e.g. length of prediction horizon) is also studied. The comparison results demonstrate that the proposed approach has approximately a 6% improvement in fuel economy over the rule-based one, and it can achieve over 98% of the fuel optimality of DP in typical working conditions. To show the advantage of the proposed MPC and its robustness under large disturbances, 40% white noise has been added to the typical working condition. Simulation results show that an 8% improvement in fuel economy is obtained by the proposed approach compared to the rule-based one.

Index Terms—Series hybrid electric tracked bulldozer, Energy management strategy, Model predictive control, Rule-based, Dynamic programming, Robustness

I. INTRODUCTION

Construction vehicles, such as bulldozers, play a significant role in modern society. The increasing reliance on construction vehicles brings serious adverse impacts such as unsustainable energy use and poor air quality. Recently, hybrid electric construction vehicles have appeared. Caterpillar produced the first hybrid electric tracked bulldozer, D7E, in March 2008. Compared to traditional models, D7E's CO and NO_x emissions were reduced by approximately 10 and 20 percent, respectively. The D7E model can improve fuel economy by 25%. In this

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paper, a new HETB composed of an engine-generator, two drive motors, and an ultracapacitor pack is put forward. The powertrain topology of the HETB is shown in Fig.1. This HETB uses an integrated controller to manipulate two separate motors on the two sides. The added electric motors and ultracapacitors provide more flexibility to meet power demands and achieve minimal fuel consumption [1]. The performance or fuel economy of the HETB is heavily dependent on its energy management strategy, which uses a supervisory controller that can coordinate the energy flow between different energy sources and enhance the overall efficiency of the powertrain [2].

Recently, numerous energy management strategies have been reported and applied to hybrid electric vehicles (HEVs) [3], [4], [5], [6], and these strategies can be divided into four classes [7]. The first type refers to the numerical optimization method, where the entire or partial drive cycle is required and the global or local optima is found numerically; this type includes the DP [8],[9],[10], MPC [11],[12] and stochastic DP [13]. DP provides a globally optimal solution and is mainly employed as a good benchmark for optimality comparison [14]. In the literature [6], authors firstly propose a novel correctional DP-based energy management strategy that takes characteristics of the drive cycle and hybrid powertrain into consideration to realize the significant improvement of fuel economy and at the same time to ensure drivability during slope conditions. The second class represents the analytical optimization method including Pontryagin's minimum principle and the Hamilton-Jacobi-Bellman equation [15]. The third type is the equivalent consumption minimization strategy (ECMS) [16], which decides the optimal power split ratio between different energy sources at each step [17],[18]. Furthermore, the ECMS method does not require future driving information as it solves an instantaneous optimization problem. Given a proper equivalent factor, ECMS could potentially achieve sub-optimal fuel economy [19]. Nevertheless, it is nontrivial to tune the equivalent factor, and ECMS cannot produce globally optimal performances. ECMS is able to adjust the factor via an adaptive ECMS as long as the future driving information can be identified online to achieve better fuel economy [20], [21]. The fourth category employs fuzzy logic, heuristic rules, and neural networks for energy management strategy design [22], [23].

Nomenclature			
F_E	external travel resistance, N	h_p	bulldozer average cutting depth, m
F_T	operating resistance, N	μ_1	friction coefficient among soil particles
F_c	compaction resistance, N	μ_2	friction coefficient between the soil and bulldozing plate
F_b	bulldozing resistance, N	Vol	the soil volume in front of the bulldozing plate, m
G	vehicle's weight, N	θ	Slope, °
b	width of the track, m	k_s	soil loose degree coefficient
L'	length of the track, m	k_y	cutting force per unit area when the plate is penetrated into the soil, MPa
c	soil cohesion coefficient, KPa	k_m	soil fullness degree coefficient
ψ	soil internal friction angle, °	α_0	natural slope angle of the soil, °
k	soil deformation modulus, KN/m ⁿ⁺²	X	bulldozing plate worn length contacting the ground, m
n	soil deformation index	δ	cutting angle of the bulldozing plate, °
Z	track's amount of sinkage, m	N_e	speed of engine, rpm
γ	soil unit weight, N/m ³	P_e	engine output power, kW
N_f, N_c	soil Terzaghi coefficients of the bearing capacity	T_e	engine torque, N
F_1	cutting force, N	n_m	motor speed, rpm
F_2	pushing force of the mound ahead of the blade, N	T_m	motor output torque, N
F_3	friction resistance between the blade and ground, N	P_{uc}	output power from the ultracapacitor, kW
F_4	component of the frictional resistance in horizontal direction when the soil rises along the blade, N	P_g	generator output power, kW
B_1	bulldozer plate width, m	η_m	motor efficiency
H	bulldozer plate height, m	C	equivalent capacitance of ultracapacitor, F
k_b	cutting force per unit area, MPa	SOC	state of charge of ultracapacitor
G_1	soil weight in front of the bulldozing plate, N	SOE	state of energy of ultracapacitor

Fig.1. Configuration of the HETB.

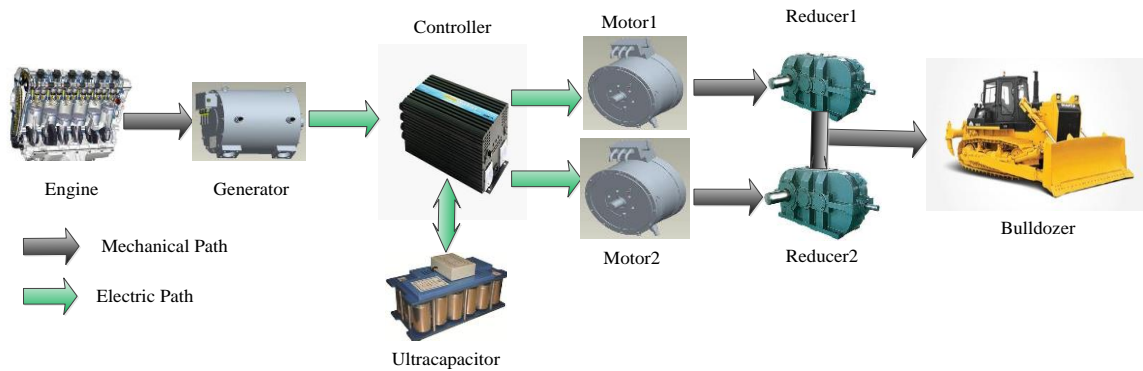


Fig.1. Configuration of the HETB.

1 The MPC is prevalent and widely employed in HEVs
 2 nowadays as an effective approach to deal with
 3 multivariable constrained control problems, and this
 4 strategy can be treated as a tradeoff between DP and
 5 ECMS. Currently, different kinds of MPCs are widely
 6 utilized because of their ability to deal with multivariable
 7 constrained problems and their potential for the real-time
 8 application as a receding horizon control strategy.
 9 Meanwhile, the MPC has also shown its potential for
 10 application in HEVs [24], [25], [26], [27], [28]. An MPC
 11 solves an energy management problem at every time instant
 12 by quadratic programming [29], nonlinear programming
 13 [30], Pontryagin's minimum principle [31], and stochastic
 14 DP [32]. In [33], a stochastic MPC was designed for a series
 15 HEV, where a Markov chain was used to model the future
 16 power demand. Its performance was compared to that of a
 17 prescient MPC with a fully known power demand and a
 18 frozen-time MPC using a constant power demand in the
 19 prediction horizon to demonstrate its fuel economy in a
 20 condition similar to the ideal condition (prescient MPC).

TABLE I
BASIC VEHICLE PARAMETERS

Component	Parameters	Quantity
Diesel Engine	maximum power	172kW/1800rpm
	maximum torque	1087Nm/1300rpm
Motor	maximum power	105kW
	rated power	75kW
	maximum torque	800Nm
	rated torque	500Nm
	maximum speed	6000rpm
	rated speed	1430rpm
Generator	maximum power	180kW
	rated power	175kW
	maximum torque	1010Nm
	rated torque	980Nm
Ultracapacitor	maximum speed	2200rpm
	rated speed	1700rpm
	capacity	2.4F
Vehicle	voltage	600V
	curb weight	28000kg
	track width	0.61m
	track length	3.05m
	drive wheel radius	0.46831m

1 Literature [34] developed an MPC for energy management
 2 with the capability to account for the uncertainty caused by
 3 traffic, destination, and weather. A modified k-nearest
 4 neighbor regressor was utilized to generate weighted
 5 samples of the upcoming drive cycle by feature matching
 6 the current state to historical states, and subsequently, an
 7 MPC was developed based on the obtained information.

8 In this paper, the MPC method is used to arrive at an
 9 effective energy management system for HETBs. HETBs
 10 are mainly different from road electric hybrid vehicles in
 11 working and driving conditions. Unlike HEVs, HETBs'
 12 power demands change dramatically between the
 13 soil-cutting stage and the no-load stage under a specific
 14 drive cycle. Consequently, the application of MPC strategy
 15 in HETBs is more complicated than that in HEVs. Besides,
 16 the drive cycle changes sharply according to the ground
 17 characteristic. Thus, the robustness of the HETB is more
 18 important than that of an HEV.

19 Three scenarios are utilized to develop the energy
 20 management controller using the MPC. The first scenario is
 21 extracted from typical working conditions of the bulldozer.
 22 The optimal solution over a typical drive cycle is obtained
 23 by achieving the maximal fuel economy and then
 24 comparing this to the results from using rule-based and DP
 25 strategies. The effect of the MPC parameters (*e.g.* length of
 26 prediction horizon) is also investigated. The comparison
 27 indicates that the proposed approach is robust to drive cycle
 28 disturbances and provide much better fuel economy over
 29 rule-based strategies. It is also indicated that the proposed
 30 MPC power management can achieve over 98% of the fuel
 31 optimality of DP without any knowledge of the changes in
 32 drive and working conditions.

33 The paper is organized as follows: In Section II, the
 34 HETB model is provided; the MPC is developed in Section
 35 III; the other two power management strategies are
 36 provided in the next section; the simulation results under
 37 three scenarios are compared to the rule-based strategy and
 38 the optimal solution calculated by DP in Section V; finally,
 39 comments and future work are discussed.

40 II. SERIES HETB POWERTRAIN MODEL

41 A. System Configuration

42 The vehicle studied is an SD-24 tracked bulldozer from
 43 Shantui Construction Machinery Co., Ltd, and its
 44 powertrain configuration can be seen from Fig.1. The series
 45 hybrid power system is composed of a diesel engine
 46 (175kW), an ultracapacitor pack, a permanent magnet
 47 generator (175/180 kW), two motor drive systems (75/105
 48 kW), and two tracks. A 2.4F ultracapacitor pack is utilized
 49 as an energy storage system. The integrated controller is
 50 developed and used to coordinate the power flow of the
 51 entire powertrain. Specifications of this bulldozer are given
 52 in Table I.

53 The HETB is modeled in SIMULINK, as shown in Fig.2.
 54 For more information regarding this model, please refer to
 55 [35].

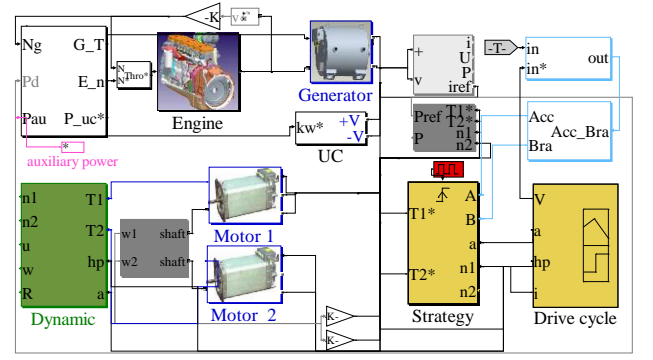


Fig.2 HETB model in SIMULINK

56 B. The Vehicle Model

57 Differing from road vehicles, the bulldozer's major
 58 external forces that are exerted on the two tracks along the
 59 heading direction include the external travel resistance F_E
 60 and the operating resistance F_T . The aerodynamic drag and
 61 the acceleration resistance are neglected since the bulldozer
 62 has a low velocity [36], [37].

63 The external travel resistance F_E is caused by the vertical
 64 deformation of the soil under the anterior track of the
 65 bulldozer when driving. It mainly results from the energy
 66 consumption of soil compaction and the effects of
 67 bulldozing resistance can be shown as [38]:

$$68 \quad F_E = F_c + F_b \quad (1)$$

$$69 \quad F_c = \frac{2b}{(n+1)k^n} \left(\frac{G}{2bL} \right)^{\frac{n+1}{n}} \quad (2)$$

$$70 \quad F_b = \gamma Z^2 b K_\gamma + 2bZcK_{pc} \quad (3)$$

71 where,

$$72 \quad Z = \left(\frac{G}{2bLk} \right)^{\frac{1}{n}} \quad (4)$$

$$73 \quad K_\gamma = \left(\frac{2N_\gamma}{\tan \psi} + 1 \right) \cos^2 \psi \quad (5)$$

$$74 \quad K_{pc} = (N_c - \tan \psi) \cos^2 \psi \quad (6)$$

75 The operating resistance F_T is shown as the following
 76 [39]:

$$77 \quad F_T = F_1 + F_2 + F_3 + F_4 \quad (7)$$

$$78 \quad F_1 = 10^6 B_1 h_p k_b \quad (8)$$

$$79 \quad F_2 = \frac{V \gamma \mu_1 \cos \theta}{k_s} \quad (9)$$

$$80 \quad F_3 = 10^6 B_1 X \mu_2 k_y \quad (10)$$

$$81 \quad F_4 = G_i \mu_2 \cos \delta^2 \cos \theta \quad (11)$$

$$82 \quad Vol = \frac{B_1 (H - h_p)^2 k_m}{2 \tan \alpha_0} \quad (12)$$

83 By combining (1) ~ (12), the vehicle's power
 84 requirements for the powertrain, P_{req} , can be formulated as:

$$85 \quad P_{req} = (F_E + F_T) V \quad (13)$$

86 where V is the bulldozer's speed along the longitudinal
 87 direction.

1 C. The Engine Model

2 The experimental approach is adopted to model the
3 engine, and the engine's dynamic characteristics are
4 neglected. The engine fuel consumption is represented by a
5 function of the mechanical power and crankshaft speed,
6 both of which were identified from experimental data as
7 shown in Fig. 3.

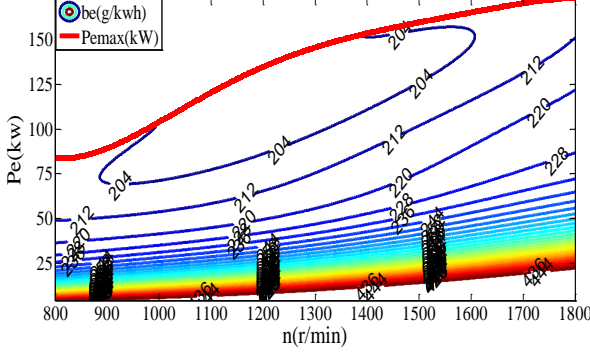


Fig.3. Fuel consumption map of the diesel engine

8 Assuming that engine is able to operate at the fixed
9 speed, the fuel consumption B_e (g/s) is a function with
10 respect to the mechanical power, P_e :

$$11 \quad B_e = B_e(P_e) \quad (14)$$

12 The engine is constrained to operate within its limits:

$$13 \quad \begin{aligned} N_{e,\min}(t) &\leq N_e(t) \leq N_{e,\max}(t); \\ P_{e,\min}(t) &\leq P_e(t) \leq P_{e,\max}(t); \\ T_{e,\min}(t) &\leq T_e(t) \leq T_{e,\max}(t); \end{aligned} \quad (15)$$

14 where $N_{e,\min}(t)$ and $N_{e,\max}(t)$ represent the lower and upper
15 limit of engine speed at time t, respectively; $P_{e,\min}(t)$ and
16 $P_{e,\max}(t)$ are the limits of the output power, respectively;
17 whereas, $T_{e,\min}(t)$ and $T_{e,\max}(t)$ are the minimum and
18 maximum engine torque at time t, respectively.

19 D. The Generator and Motor Models

20 The generator and motor efficiency characteristics are
21 represented by a non-linear 3-D Map with respect to torque
22 and speed using experimental data. The generator
23 efficiency map is provided in Fig.4, and the motor
24 efficiency map is indicated in Fig.5. The motor efficiency
25 η_m at the operation point (n_m, T_m) is calculated according to
26 the following correlation:

$$27 \quad \eta_m(n_m, T_m) = f(n_m, T_m) \quad (16)$$

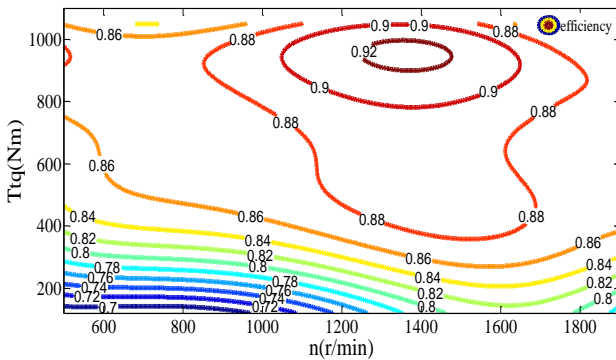


Fig.4 Generator efficiency map

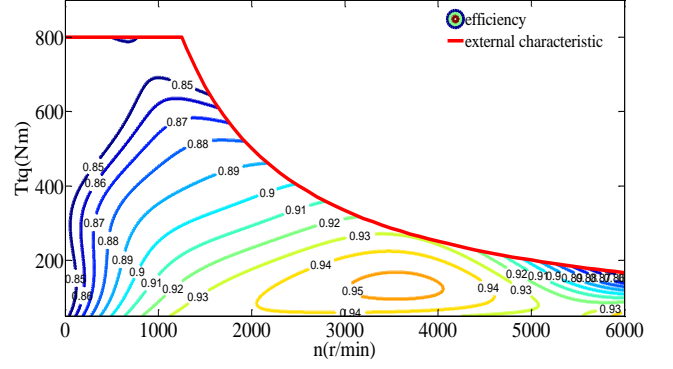


Fig.5 Motor efficiency map

28 E. The Ultracapacitor Model

29 The ultracapacitor pack is composed of several units in
30 both parallel and series modes. Each unit can be modeled as
31 a resistor in series with a capacitance. The resistance
32 models the electrolyte losses, while the capacitance
33 calculates ion accumulation. The model of the entire
34 ultracapacitor pack can be denoted by:

$$35 \quad P_{uc}(t) = V_L(t) \cdot I_{cap}(t) \quad (17)$$

$$36 \quad V_{cap}(t) = -\frac{1}{C} I_{cap}(t) \quad (18)$$

$$37 \quad SOC(t) = \frac{Q(t)}{Q_{\max}} = \frac{CV_{cap}(t)}{CV_{\max}} = \frac{V_{cap}(t)}{V_{\max}} \quad (19)$$

$$38 \quad SOE(t) = \frac{E(t)}{E_{cap}} = \frac{\frac{1}{2} CV_{cap}(t)^2}{\frac{1}{2} CV_{\max}^2} = \frac{V_{cap}(t)^2}{V_{\max}^2} = SOC(t)^2 \quad (20)$$

39 where V_L is the terminal voltage; V_{cap} is the voltage across
40 the equivalent capacitance; V_{\max} is the ultracapacitor's
41 maximum voltage; I_{cap} is the current; Q_{\max} is the maximum
42 acceptable amount of capacity; $Q(t)$ is the amount of charge
43 stored in the capacitance; E_{cap} is maximum energy capacity;
44 and $E(t)$ represents the amount of energy stored in the
45 capacitance.

46 The relationship among the differential of SOE , the
47 maximum energy capacity, and the ultracapacitor power is
48 shown in (21). Since the problem is modeled by the power
49 balance equations, choosing the SOE as the control variable
50 for the HETB is more natural. The dynamic equation of the
51 SOE variation is shown as:

$$52 \quad \dot{SOE}(t) = \begin{cases} -\frac{1}{\eta_{cap}} \frac{P_{uc}(t)}{E_{cap}} & \text{if } P_{uc}(t) \geq 0 \text{ (discharge)} \\ -\eta_{cap} \frac{P_{uc}(t)}{E_{cap}} & \text{if } P_{uc}(t) < 0 \text{ (charge)} \end{cases} \quad (21)$$

53 where η_{cap} is the ultracapacitor's efficiency.

54 The power balance model for the electrical summation
55 node is shown in Fig.6, where the relationship among the
56 power from the genset, the electric motor, and the
57 ultracapacitor is described as:

$$58 \quad P_{uc} = P_{gen,e} + P_{req} \quad (22)$$

$$P_e = \frac{P_{gen,e}}{\eta_g} = \frac{P_{uc} - P_{req}}{\eta_g} \quad (23)$$

where P_{req} is the power requirements from the powertrain; $P_{gen,e}$ denotes the electric power from the genset; and η_g is the generator efficiency.

From (22), the following constraints on P_{uc} are derived:

$$P_{req}(t) - P_{gen,e,max} \leq P_{uc}(t) \leq P_{req}(t) - P_{gen,e,min} \quad (24)$$

Furthermore, P_{uc} and the SOE must be satisfied together with the physical constraints:

$$P_{uc,min}(t) \leq P_{uc}(t) \leq P_{uc,max}(t) \quad (25)$$

$$SOE_{min} \leq SOE(t) \leq SOE_{max} \quad (26)$$

where $P_{gen,e,max}$ represents the maximum electric power from the genset; $P_{gen,e,min}$ refers to the minimum power; $P_{uc,max}$ is the maximum output power of ultracapacitor; $P_{uc,min}$ is the minimum output power; SOE_{max} denotes the maximum state of energy; and SOE_{min} is the minimum state of energy.

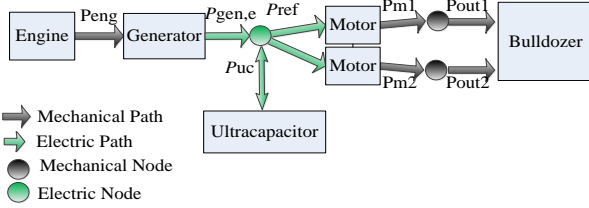


Fig.6. Power flow of the HETB.

III. MPC DEVELOPMENT FOR SERIES HETB

As an optimal control method, the MPC originated as a control technique in the chemistry industry. It is characterized by its slow dynamics, which provides enough time for optimization calculations. According to the HETB model developed in the previous section, the model predictive controller can be developed using the following equations:

$$\dot{x}_1 = -\frac{u(t)}{E_{cap}} \quad (27)$$

$$\dot{x}_2 = \dot{B}_e(P_e) = \dot{B}_e\left(\frac{P_{uc} - P_{req}}{\eta_g}\right)$$

where $x_1=SOE$ and $x_2=B_e$ denotes the fuel consumption; while, $u=P_{uc}$ represents the control input.

The vectors of states, control inputs, measured inputs, as well as the outputs are defined as:

$$x = \begin{bmatrix} SOE \\ B_e \end{bmatrix}, u = P_{uc}, v = P_{req}, y = \begin{bmatrix} SOE \\ B_e \end{bmatrix} \quad (28)$$

The linearized and discretized model of the system becomes:

$$\begin{cases} x(k+1) = A(k)x(k) + B_u(k)u(k) + B_v(k) \\ y(k) = C(k)x(k) \end{cases} \quad (29)$$

In this equation,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; B_u(k) = \begin{bmatrix} -\frac{1}{E_{cap}} \\ -m_1 \end{bmatrix}; B_v(k) = \begin{bmatrix} 0 \\ m_1 * P_{req} + m_2 \end{bmatrix}; C(k) = \begin{bmatrix} 10 \\ 0 & 1 \end{bmatrix};$$

The cost function to be minimized can be described by:

$$\begin{aligned} \min_{u_0, u_1, \dots, u_{N-1}} J & \\ &= \min_{u_0, u_1, \dots, u_{N-1}} \sum_{i=1}^{N-1} \left[w_{i+1}^y \|y(k+i+1|k) - y_{ref}(k+i+1|k)\|^2 + w_i^u \|u(k+i|k)\|^2 \right] \\ &= \min_{u_0, u_1, \dots, u_{N-1}} \sum_{i=1}^{N-1} \left[y(k+i+1|k) - y_{ref}(k+i+1|k) \right]^T Q \\ & \quad \left[y(k+i+1|k) - y_{ref}(k+i+1|k) \right] + u(k+i|k)^T R u(k+i|k) \end{aligned} \quad (30)$$

s.t.

$$y_{min} \leq y(k) \leq y_{max}, k=0,1,\dots,N-1$$

$$u_{min} \leq u(k) \leq u_{max}, k=0,1,\dots,N-1$$

In the above equation, N is the prediction horizon length; w^y and w^u refers to the weights for the output y and control input u , respectively.

The objective function has been formulated for the energy management problem of the HETB. The main objective is to achieve optimal fuel economy by tracking the SOE reference value. The SOE reference trajectory is obtained from the dynamic programming (DP) optimization and the fuel consumption's reference trajectory is taken as zero. The constraints on the control effort involved are imposed by enforcing (24) and (25) at each time step. The state penalty Q and the input penalty R are:

$$Q = \begin{bmatrix} 1000000 & 0 \\ 0 & 1 \end{bmatrix}; R = 10$$

The objective function is transferred into a quadratic form with regard to the control input. The trajectory of future states will be obtained by the discrete model as the prediction horizon length is N [40]:

$$\begin{aligned} \underbrace{\begin{bmatrix} x(k+1) \\ x(k+2) \\ \vdots \\ x(k+N) \end{bmatrix}}_{\bar{x}} &= \underbrace{\begin{bmatrix} A \\ A^2 \\ \vdots \\ A^N \end{bmatrix}}_{S^x} x(k) + \underbrace{\begin{bmatrix} B_u & 0 & \dots & 0 \\ AB_u & B_u & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ A^{N-1}B_u & A^{N-2}B_u & \dots & B_u \end{bmatrix}}_{S^u} \underbrace{\begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+N-1) \end{bmatrix}}_{\bar{u}} \\ &+ \underbrace{\begin{bmatrix} B_v(k) \\ B_v(k) + B_v(k+1) \\ \vdots \\ B_v(k) + B_v(k+1) + \dots + B_v(k+N-1) \end{bmatrix}}_v \end{aligned} \quad (31)$$

$$\underbrace{\begin{bmatrix} y(k+1) \\ y(k+2) \\ \vdots \\ y(k+N) \end{bmatrix}}_{\bar{y}} = \underbrace{\begin{bmatrix} C & 0 & 0 & 0 \\ 0 & C & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & C \end{bmatrix}}_{C^*} \bar{x}$$

The convex quadratic objective function only with respect to the input will be obtained by inserting (31) into the original objective function shown in (30) and neglecting the constant term:

$$J(x_0, u_0) = \frac{1}{2} \bar{U}^T H \bar{U} + F^T \bar{U} \quad (32)$$

$$H = 2(C^x S^u)^T \bar{Q}(C^x S^u) + \bar{R}$$

$$F = 2(C^x S^u)^T \bar{Q}(C^x S^u - \bar{Y}_{ref})$$

s.t.

$$\bar{U} \geq \max(\bar{U}_{\min}(U), \bar{U}_{\min}(\bar{U}), \bar{U}_{\min}(X))$$

$$\bar{U} \leq \min(\bar{U}_{\max}(U), \bar{U}_{\max}(\bar{U}), \bar{U}_{\max}(X))$$

1 where the Hessian matrix H is symmetric and positive or
 2 semi-positive definite and F is the gradient vector. \bar{Q} , \bar{R}
 3 and \bar{Y}_{ref} should be reformulated according to the prediction
 4 horizon length N based on Q , R , and Y_{ref} . The updated
 5 constraints of the increment of the control can be found by
 6 the reformulation of (32) and the constraints shown in (30).
 7 For example, the constraints of the states can be applied to
 8 \bar{U} as $\bar{U}_{\max}(X)$.

9 The energy management problem is solved by an open
 10 source solver, qpOASES [41]. The optimal control input
 11 sequence $u_0, u_1, u_2 \dots u_{N-1}$ is obtained from the solver
 12 qpOASES, and the first element of this trajectory u_0 is
 13 applied to the plant model of the HETB. The updated value
 14 of the state is obtained in the subsequent step. The receding
 15 control strategy is implemented by repeating this procedure
 16 during subsequent time steps. The explicit expression of the
 17 quadratic programming is not reported here for the sake of
 18 brevity.

19 IV. RULE-BASED AND DP-BASED ENERGY MANAGEMENT 20 STRATEGIES

21 In this paper, three energy management strategies have
 22 been designed in order to study the potential fuel economy
 23 of an HETB: rule-based strategy, DP, and MPC.

24 A. The Rule-based strategy

25 Utilizing a set of rules is the most popular and easiest
 26 method of implementing supervisory control in an HEV
 27 and deciding on the power split ratio between the engine
 28 and the other energy storage system [42]. The parameters of
 29 a rule-based controller are usually obtained from the
 30 powertrain modeling and simulation, possibly by using
 31 optimization techniques. In this study, the rule-based
 32 approach is implemented as follows: the engine output
 33 power follows the power demand of the bulldozer, and the
 34 ultracapacitor acts as the auxiliary power source to supply

TABLE II
 RULE-BASED CONTROL STRATEGY

Judgment	State of the UC	Power supply
$P^* < P_{e_max}$ $SOC < SOC_{max}$	Charging	$P_g = \eta_g * P_e$ $P_{uc} = P_{dc} - P_g$
$P^* < P_{e_max}$ $SOC \geq SOC_{max}$	Not-working	$P_g = \eta_g * P_e$ $P_{uc} = 0$
$P^* > P_{e_max}$ $SOC > SOC_{min}$	Discharging	$P_g = \eta_g * P_{e_max}$ $P_{uc} = P_{dc} - P_g$
$P^* > P_{e_max}$ $SOC \leq SOC_{min}$	Not-working	$P_g = \eta_g * P_{e_max}$ $P_{uc} = 0$

35 power for the power shortage caused by the excessive load
 36 of the power demand. The SOC of the ultracapacitor and
 37 load power requirement determines the working point of
 38 the engine-generator, as shown in Table II.

39 In this table, P_{e_max} represents the engine's maximum
 40 power; P^* refers to the target demand power; P_{dc} represents
 41 the DC bus demand electric power; P_{uc} is the ultracapacitor
 42 power; and SOC_{max} and SOC_{min} are the ultracapacitor
 43 maximum and minimum state of charge, respectively.

44 B. Dynamic programming

45 Differing from the rule-based strategy, the DP algorithm
 46 usually depends on a model to provide a provably optimal
 47 control strategy by searching all state and control grids
 48 exhaustively [43], [44]. However, the DP-based approach is
 49 not suitable for real-time application since the exact future
 50 driving information is seldom known in the real world [45].
 51 Nonetheless, the DP-based strategy can provide a good
 52 benchmark for evaluating the optimality of other
 53 algorithms, which helps in ultimately perfecting real-time
 54 strategies [46], [47], [48].

55 The problem setup for the DP-based strategy requires
 56 discrete values of the control variable and a discrete-time
 57 description of the system. The procedure of DP is
 58 implemented as follows [6].

59 1) Problem Formulation

60 The state and the control variables need to be determined
 61 in order to formulate the DP. As mentioned, the state is the
 62 SOE . The control input refers to the output power of the
 63 ultracapacitor. The discrete-time model of the HETB can be
 64 expressed as:

$$65 \quad x(k+1) = f(x(k), u(k)) \quad (33)$$

66 In the above equation, $u(k)$ and $x(k)$ are the control inputs
 67 and the state variables, respectively. The sampling time is
 68 chosen as 1 second.

69 The purpose of this optimization problem is to obtain the
 70 optimal control sequence, $u(k)$, and minimize the fuel
 71 consumption over a given drive cycle. The cost function of
 72 this optimization problem is described as follows:

$$73 \quad J = \sum_{k=0}^{M-1} L(x(k), u(k)) \quad (34)$$

74 where, L means the instantaneous cost value and M is the
 75 time length of the specific drive cycle.

76 The physical constraints of state and control variables are
 77 denoted by the following inequalities to guarantee
 78 smooth/safe operation of the key components, including the
 79 engine, motor, and ultracapacitor:

$$80 \quad \begin{cases} SOC_{min} \leq SOC \leq SOC_{max}; \\ SOE_{min} \leq SOE \leq SOE_{max}; \\ N_{e_min} \leq N_e \leq N_{e_max}; \\ P_{e_min} \leq P_e \leq P_{e_max}; \\ T_{e_min} \leq T_e \leq T_{e_max}; \end{cases} \quad (35)$$

81 Furthermore, the equality constraints are used such that
 82 the HETB can satisfy load and speed requirements at all
 83 times.

84 2) Implementing Dynamic Programming

85 The main merit of DP is that it is able to deal with the

1 nonlinear problem and constraints while obtaining the
2 optimal policy. The DP problem can be described by (36)
3 and (37):

4 Step $M-1$:

$$5 \quad J_{M-1}^*(x(M-1)) = \min_{u(M-1)} [L(x(M-1), u(M-1))] \quad (36)$$

6 Step k , for $0 \leq k < M-1$:

$$7 \quad J_k^*(x(k)) = \min_{u(k)} [L(x(k), u(k)) + J_{k+1}^*(x(k+1))] \quad (37)$$

8 where $J_k^*(x(k))$ refers to the optimal accumulated cost from
9 time step t_k to the terminal; whereas, $x(k+1)$ means the state
10 at the $(k+1)$ th stage when the control variable u_k is applied
11 at the time step t_k according to (29).

12 The optimal control policy is obtained by solving the
13 above recursive equation backwards. The minimizations
14 are conducted subject to the equality constraints imposed
15 by the drive cycle and the inequality constraints shown in
16 (35).

17 V. CASE STUDY

18 In this section, the results obtained by the
19 aforementioned three energy management strategies are
20 compared and discussed in three scenarios.

21 A. Scenario 1: Typical working condition

22 In this scenario, a typical working condition is used for
23 the simulation to investigate the effect of the prediction
24 horizon length. In Fig. 7, Velocity (km/h) is the bulldozer
25 velocity and the depth (m) is soil-cut depth. The working
26 stages are described as follows: 1~4-s is the traveling stage;
27 4~16-s is the soil-cutting stage; 16~31-s is the
28 soil-transportation stage; 31~33-s is the unloading soil
29 stage, and 33~50-s is the no-load stage. Fig.8 shows the
30 power demand calculated according to the typical working
31 condition by the equations described in Section II.

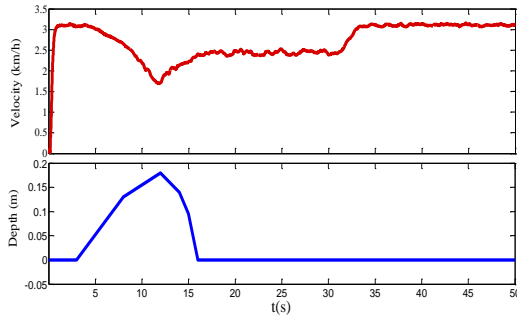


Fig.7 Typical working condition of HETB

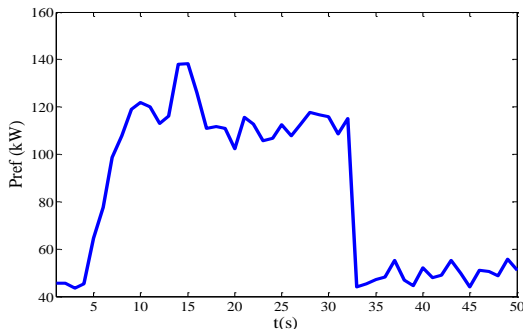


Fig.8 Power demand of the typical working condition

32 The most important MPC parameter that affects the
33 solution is the length of the prediction horizon, N , which
34 can be 2s, 4s, or 15s. Fig.9 shows the SOE profile
35 corresponding to the different lengths of prediction
36 horizons and the optimal solution obtained from the DP
37 algorithm. It can be observed that as the prediction horizon
38 increases, the MPC draws closer to the optimal solution.
39 The improvement in fuel economy is provided in Table III.
40 To compensate for the discrepancy between the initial SOE
41 and final SOE , the correction method proposed in [13] is
42 used such that the comparison can be performed. As seen
43 from Table III, the fuel consumption also decreases with an
44 increase of the receding horizon. Finally, a prediction
45 horizon of 15s will be chosen and used in the MPC
46 development in the following two scenarios.

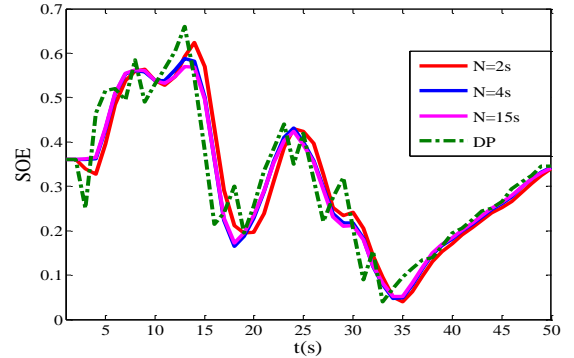


Fig.9. SOE profile with different length of prediction horizon.

47 Fig.10 shows the SOE , ultracapacitor's current, engine
48 power, and the ultracapacitor's output power. The
49 trajectories of the engine's power and the ultracapacitor's
50 power demonstrate the optimal power split between two
51 energy resources to result in minimal fuel consumption.
52 Fuel economy achieved by the MPC algorithm is compared
53 to the DP algorithm and the rule-based algorithm over the
54 same working condition shown in Fig.7. As indicated in
55 Table III, DP helps the HETB consume the minimal amount
56 of fuel, 290g. The fuel consumption of the rule-based
57 algorithm from the previous work is 313g, and its fuel
58 economy is 92.6% of the optimal one. The fuel economy of
59 the MPC algorithm is better than that of the rule-based
60 algorithm and much closer to that of the DP algorithm. An
61 additional 6% fuel economy is obtained by MPC algorithm
62 over the rule-based one. The MPC can achieve 98.6% fuel
63 optimality in relation to the optimal DP under a typical
64 driving scenario. Although DP cannot be used in real time,
65 analyzing its behavior can provide meaningful insight into
66 the possible improvement of the MPC controller.

TABLE III
FUEL CONSUMPTION COMPARISON UNDER SCENARIO 1

Control Strategy	Fuel Consumption (g)	Fuel Economy (%)	
DP	290	100	
Rule-based	313	92.6	
MPC	N=2	295.4	98.1
	N=4	294.6	98.4
	N=15	294	98.6

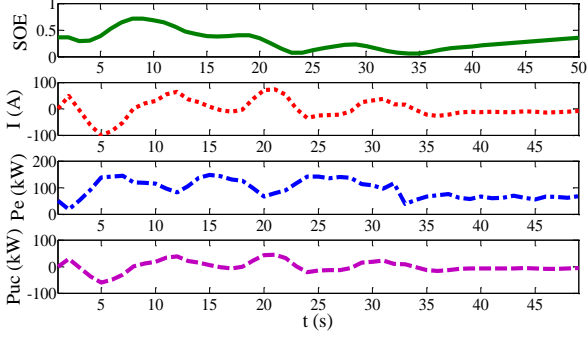


Fig.10. MPC results under scenarios 1

1 B. Scenario 2: The Working Condition under 2 Disturbances

3 In order to verify the robustness of the proposed MPC
4 strategy, a disturbance of 40% is added to the typical
5 working condition as shown in Fig.11.

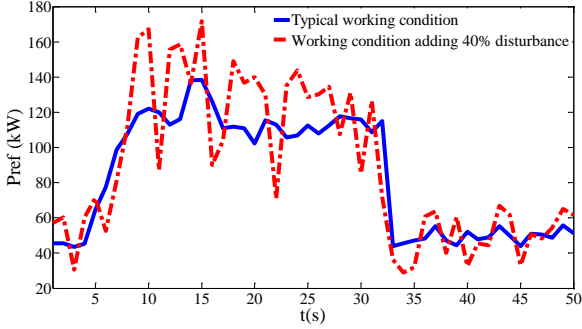


Fig.11. Power demand comparison under scenario 2

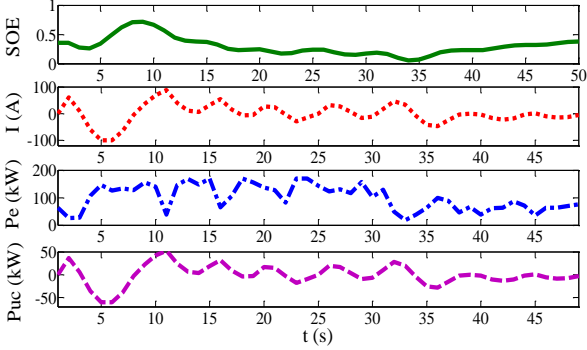


Fig.12. MPC results under scenario 2

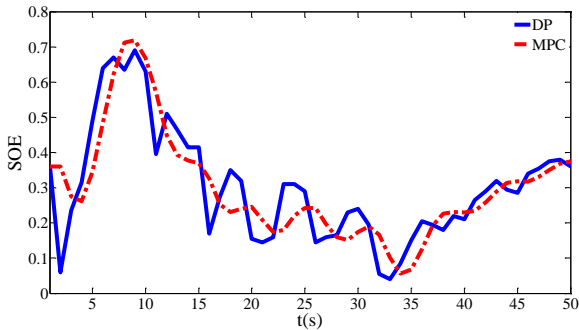


Fig.13. SOE profile comparison under scenario 2

6 The results of the system SOE , ultracapacitor's current,
7 engine power, and input P_{uc} are presented in Fig.12. Fig. 13
8 shows the comparison of the SOE between the MPC and the
9 DP under Scenario 2. The fuel consumption of the three

10 energy management strategies is shown in Table IV. The
11 MPC algorithm can achieve 98.9% fuel optimality with
12 respect to the DP benchmark under scenario 2; whereas, the
13 rule-based power management can only achieve 91%. The
14 MPC strategy can obtain an additional 8% fuel economy
15 improvement over that of the rule-based strategy. We can
16 conclude that the MPC strategy is more effective when the
17 working condition is not fully known.

TABLE IV
FUEL CONSUMPTION COMPARISON IN SCENARIO 2

Control Strategy	Fuel Consumption (g)	Fuel Economy (%)
DP	304.7	100
Rule-based	334.8	91
MPC	308	98.9

18 C. Scenario 3: The Combined Working Condition

19 Although the working condition is preset, there would be
20 uncertainties or disturbances in real applications where the
21 real working condition would distribute around the typical,
22 preset working condition. Therefore, a combined working
23 condition with a 40% disturbance is used to evaluate the
24 MPC's robustness, as shown in Fig.14. The same MPC
25 power management strategy is used for the disturbed
26 combined working conditions.

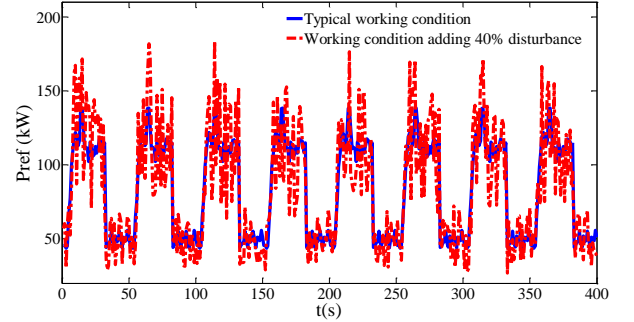


Fig.14. Power demand comparison under scenario 3

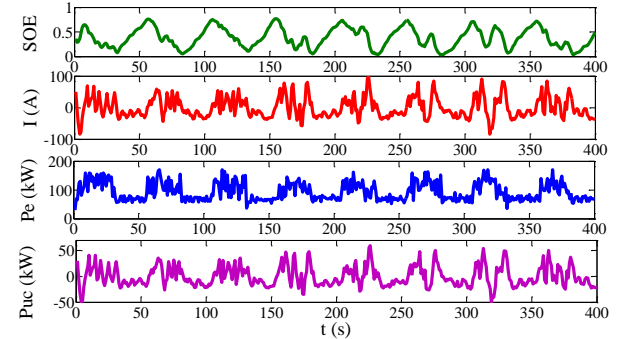


Fig.15. Power demand comparison under scenario 3

TABLE V
FUEL CONSUMPTION COMPARISON IN SCENARIO 3

Control Strategy	Fuel Consumption (g)	Fuel Economy (%)
DP	2259.5	100
Rule-based	2583.6	87.5
MPC	2376.9	95

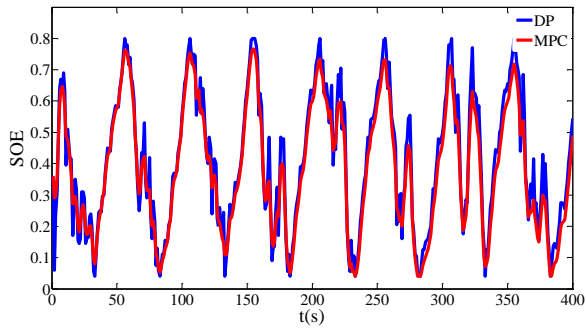


Fig. 16. SOE profile comparison under scenario 3

1 The results of the system SOE , ultracapacitor's current,
 2 engine power, and input P_{uc} are presented in Fig. 15. Fig. 16
 3 shows the comparison of the SOE between the MPC and the
 4 DP under scenario 3. The fuel consumption comparison is
 5 shown in Table V. The DP-based control strategy with the
 6 actual working condition is used to evaluate the MPC and
 7 rule-based performances in the presence of drive cycle
 8 disturbances. It can be seen from Table V that the MPC
 9 algorithm can achieve 95% fuel optimality with respect to
 10 the DP benchmark under scenario 3 while the rule-based
 11 power management can only achieve 87.5%. An additional
 12 8% fuel economy improvement is obtained from the MPC
 13 algorithm over that of the rule-based strategy.

14 The conclusion can be drawn that even under disturbed
 15 conditions, the MPC can work very well in spite of using
 16 the typical working condition for its prediction. One
 17 simulation step has the calculation time of mere
 18 milliseconds, so this proposed MPC can be used in real
 19 time. All results demonstrate that the proposed MPC is
 20 robust and applicable.

VI. CONCLUSION

22 The application of the model predictive energy
 23 management strategy of a series HETB was presented in
 24 this study. In order to develop the MPC strategy, the
 25 structure and modeling of the HETB were discussed, and
 26 the effect of the most important MPC parameters was
 27 investigated after implementation of the proposed strategy.

28 This paper also presented a comparative study between
 29 the MPC and two other strategies: 1) rule-based control
 30 strategy; 2) DP algorithm for minimizing fuel consumption.
 31 The structure and modeling of the HETB were developed
 32 first. Using this model, the formulations of three energy
 33 management strategies were presented. Simulation results
 34 showed that under the typical working condition, the fuel
 35 economy achieved with the MPC is 6% better than that
 36 achieved by the rule-based algorithm. The proposed MPC
 37 power management also demonstrated that it can achieve
 38 98% fuel optimality with respect to the DP benchmark in
 39 the typical working condition.

40 In order to verify the advantage of the MPC strategy
 41 under large disturbances, a 40% white noise was added to
 42 the typical working condition. Simulation results
 43 demonstrated that the MPC strategy can obtain an
 44 additional 8% fuel economy improvement over that of the

45 rule-based strategy under disturbed scenarios. This shows
 46 the robustness of the proposed energy management for
 47 large disturbances.

48 Further simulation and experimental investigations are
 49 underway to test and verify these quantitative results.
 50 Future work will focus on real-world cases to evaluate the
 51 proposed power management strategy and to make it more
 52 robust under all working and driving conditions.

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